X-Ray polarimetry as a probe of magnetic turbulence in supernova remnants. IXPE perspective.

Andrei Bykov, Iurii Uvarov
Ioffe Institute, St.Petersburg, Russia
Particle diffusion on magnetic fluctuations is essential for particle confinement in the acceleration region.

Particle acceleration and magnetic field generation processes are highly depended on each other. Fermi acceleration is highly nonlinear process.

Accelerated particles penetrate upstream and produce additional pressure that can eventually modify fluid velocity in the upstream.

Diffusive shock acceleration time
(Berezhko, Krymskii 1988)

\[ \tau_a \approx \frac{3}{U_1 - U_2} \left( \frac{D_1}{U_1} + \frac{D_2}{U_2} \right) \sim \frac{6D}{U^2} \]

\[ D = \frac{crH}{3} = \frac{E_{c}}{3eH} \]
Maximal energies of electrons accelerated by DSA

DSA produces a power law particle spectrum in the broad energy range. The maximum energies can be limited by the acceleration time, particle escape and synchrotron losses. The synchrotron limit of the maximum possible achieved energy can be easily estimated for electrons.

\[
\frac{dE_{keV}}{dt} = -1.65 \cdot 10^{-5} \frac{H_G^2}{8\pi} \left( \frac{E_{keV}}{511} \right)^2
\]

\[
\tau_s = 4 \cdot 10^{11} \frac{1}{H_G^2 E_{keV}}
\]

The estimation for maximum energy can be obtained if one suggests \( \tau_a = \tau_s \)

\[
E_{max, keV} \approx \sqrt{2} \frac{U_{cm/s}}{\sqrt{H_G}}
\]
Maximal energies of synchrotron photons

Maximum of the synchrotron emission of monoenergetic electron with energy $E$ is produced on frequency $\nu_m = 0.29\nu_c = 0.069(eH_/mc)(E/mc^2)^2$. On higher energies spectrum falls exponentially.

$$\nu_{Hz} = 0.29 \cdot \frac{3eH}{4\pi m_e c} \left( \frac{E}{m_e c^2} \right)^2 = 4.7 H_G E_{keV}^2$$

$$\nu_{max, keV} = 4 \cdot 10^{-17} U_{cm/s}^2$$

For $U_{cm/s} = 4x10^8$ cm/s we get $\nu_{max, keV} = 6$ keV.

Maximal energies of the synchrotron photons in the one zone model is not sensitive to the magnetic field.
SNR Tycho.

The left image is a 4-6 keV Chandra intensity map (credit NASA/CXC/SAO, K.A. Eriksen et al. 2011). The right image is a 1-8 keV Chandra image.

\[
\begin{align*}
R & \approx 2.3 \text{ кпс} \\
V & \approx 2200 \div 3500 \text{ км/с} \\
D & \approx 8' \\
T & \approx 440 \text{ лет}
\end{align*}
\]
Synchrotron maps.

In order to simulate synchrotron images we need to know an electron distribution function together with a turbulence magnetic field realization.

To obtain SNR synchrotron emission maps we calculated stochastic magnetic field in a box intersecting with SNR shell. We made simulations for isotropic and anisotropic magnetic turbulence cases. The size and other parameters of the model remnant were chosen to match the parameters for Tycho SNR.

On the left the relative position of the SNR remnant and the box where stochastic magnetic field was calculated is shown. The line of sight towards observer is along the Oz axis. On the right the observed part of the shell is shown with the effect of projection along the line of sight taken into account.
An electron distribution function was calculated according to work Bykov & Uvarov 1999 with added accounting for synchrotron radiation losses. The diffusion limit of a kinetic equation was used:

\[ D(p) \frac{\partial^2 f}{\partial x^2} - u(x) \frac{\partial f}{\partial x} + \frac{p}{3} \frac{\partial f}{\partial p} \frac{\partial u}{\partial x} - \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 L(p) f \right) = 0 \]

\( D(p) = \eta c r_c(p)/3 \) is a particle diffusion coefficient (which equals to Bohm diffusion coefficient if \( \eta = 1 \)), \( r_c(p) = p c / eH \) is a Larmor radius, \( L(p) = dp/dt \) is a function of energy losses. On high electron energies we can take into account only synchrotron losses and

\[ L(p) = \frac{dp}{dt} = -\frac{32\pi}{9} \left( \frac{e^2}{m_e c^2} \right)^2 \left( \frac{H_{av}^2}{8\pi} \right) \left( \frac{p^2}{m_e^2 c^2} \right) \]

An electron distribution function in the vicinity of the SNR shock wave is shown for different values of distance from the shock. Negative values correspond to the upstream (\( r > R_{\text{SNR}} \)) region, while positive values correspond to the downstream (\( r < R_{\text{SNR}} \)) region. The magnetic field \( H = 5 \times 10^{-5} \) G used in synchrotron losses calculations was taken to be equal to the mean square value of the magnetic field.
Simulation of an isotropic stochastic magnetic field.

According to our previous works (Bykov et al. 2008, Bykov et al. 2009) we use a method suggested in Giakalolone & Jokipii 1999. We consider summation of a big number of harmonics with random wave vectors and phases.

$$B(r,t) = \sum_{n=1}^{N_m} \sum_{\alpha=1}^{2} A^{(\alpha)}(k_n) \cos(k_n \cdot r - \omega_n(k_n) \cdot t + \phi_n^{(\alpha)}) ,$$

Two orthogonal polarizations $A^{(\alpha)}(k_n)$ (a=1,2) of magnetic field are both orthogonal to the wave vector ($A^{(\alpha)}(k_n) \perp k_n$) to fulfill zero divergence condition $\text{div } B=0$. We make a summation of independent plane waves with wave vector amplitudes $A^2(k) = C <B^2> k^{-(\delta+2)}$. This corresponds to a spectral energy density of turbulent magnetic field $W(k) \sim k^{-\delta} \sim k^2 A^2(k)$ where $\delta$ is an index of power spectrum.

$<B^2>$ is an average square of a magnetic field. The stochastic properties of an isotropic field are $<B_x^2>=<B_y^2>=<B_z^2>=<B^2>/3$. 
Anisotropic stochastic magnetic field.

We consider a case of cylindrically symmetric turbulence when an average values have the following properties: $\langle B_\perp^2 \rangle = q \langle B_{\parallel}^2 \rangle = q \langle B^2 \rangle / (q+1)$.

The previously described method can be extended to this case. Let's consider one particular magnetic harmonic. For the 1st polarization we chose a direction of magnetic field $B_1$ to lie in a plane made by the symmetry axis and the wave vector (fig. 3). For the 2nd polarization the direction of magnetic field $B_2$ will be orthogonal both to the symmetry axis and the wave vector. The 2nd polarization gives contribution only to the value of $B_\perp$ while the 1st polarization gives contribution both to $B_\perp$ and $B_{\parallel}$. If we increase an amplitude of the 2nd polarization we get an anisotropic magnetic field with desired properties.

Direction of the symmetry axis $||$, the wave vector and magnetic field polarizations of one particular harmonic are shown. Left and right panels schematically show the isotropic and the anisotropic turbulence cases respectively.

It appears that in the anisotropic case all modeling remains the same as in isotropic case but the amplitudes of harmonics should be slightly different: $A_1^2 = (1-a)A^2(k)$, $A_2^2 = (1+a)A^2(k)$, $a > 0$, $q = (2+a)/(1-a)$. 
Formation of anisotropic turbulence.

If \( B_n = 0 \), then \( B_{t,up} u_{up} = B_{t,down} u_{down} \).

Initially isotropic upstream turbulent magnetic field should be anisotropic in the downstream. \( q > 2 \).

In the case of plasma cascade the direction of a large scale magnetic field is a direction of symmetry axis. In a weak cascade only perpendicular to this direction components of the wave vectors increase. In a strong cascade there is also dependence on \( k_\parallel \). (Sridhar & Goldreich 1994, 1995). \( q < 2 \).

If large scale field is directed radially then a most probable direction of a turbulent field evolved due to cascading also should be directed radially. This case is shown on the right panel.
Properties of a radiation field can be described by 4 Stokes parameters: I, Q, U, V. These parameters for synchrotron emission of isotropic ultrarelativistic electron distribution with energy spectrum $f_e(E)$ can be represented as (Ginzburg V.L., Syrovatskii S.I. 1965): $V=0$,

$$I(\nu) = \int I(\nu, r) dr = \frac{\sqrt{3}e^3}{mc^2} \int dr dE \frac{\nu}{\nu_c} B_\perp (r) f_e (E, r) \int_0^\infty K_{5/3}(\eta) d\eta,$$

$$Q(\nu) = \int Q(\nu, r) dr = \frac{\sqrt{3}e^3}{mc^2} \int dr dE \frac{\nu}{\nu_c} \cos (2\chi) B_\perp (r) f_e (E, r) K_{2/3} \left( \frac{\nu}{\nu_c} \right),$$

$$U(\nu) = \int U(\nu, r) dr = \frac{\sqrt{3}e^3}{mc^2} \int dr dE \frac{\nu}{\nu_c} \sin (2\chi) B_\perp (r) f_e (E, r) K_{2/3} \left( \frac{\nu}{\nu_c} \right),$$

$$\nu_c = \frac{3eB}{4\pi mc^2} \gamma^2, \int dEd\Omega_E \cdot f_e (E, r) = 4\pi \int dE \cdot f_e (E, r) = n(r).$$

$\chi$ is an angle between a chosen direction in a plane perpendicular to wave vector $\mathbf{k}$ and a major axis of the polarization ellipse. Degree of polarization can be obtained using the formula:

$$\Pi_\nu = \frac{\sqrt{Q^2_\nu + U^2_\nu + V^2_\nu}}{I_\nu}$$
The main axis of the polarization ellipse of emitted synchrotron radiation is perpendicular locally to the direction of the magnetic field projection $\mathbf{B}_\perp$. If we choose to measure rotation angle of the polarization ellipse $\chi$ from the $-\mathbf{Ox}$ axis then $\cos(\chi) = \frac{B_{\perp y}}{B_{\perp}}$ and $\sin(\chi) = \frac{B_{\perp x}}{B_{\perp}}$. So we get:

$$Q(\nu) = \frac{\sqrt{3}e^3}{m c^2} \int dr dE \frac{\nu}{\nu_c} B_\perp(r) f_e(E, r) K_{2/3} \left( \frac{\nu}{\nu_c} \right) \frac{B_{\perp y}^2 - B_{\perp x}^2}{B_{\perp}^2}$$

$$U(\nu) = \frac{\sqrt{3}e^3}{m c^2} \int dr dE \frac{\nu}{\nu_c} B_\perp(r) f_e(E, r) K_{2/3} \left( \frac{\nu}{\nu_c} \right) \frac{2B_{\perp y} \cdot B_{\perp x}}{B_{\perp}^2}$$

A polarization ellipse of the emitted radiation and a possible definition of the angle $\chi$ is shown. The view is in the direction from the observer towards the emission region. The transverse to the line of sight part of the magnetic field in the emission region $\mathbf{B}_\perp$ is also shown together with directions of unit polarization vectors $\mathbf{e}_1$, $\mathbf{e}_2$ directed along the axis of the polarization ellipse.
In a case of an isotropic stochastic magnetic field or an anisotropic field with axial symmetry which symmetry axis coincides with the direction to the observer the x and y field projections have equal properties and $<Q>=0$, $<U>=0$. It means that the average polarization is suppressed and after averaging over an ensemble of all possible stochastic field realizations Q and U should be equal to zero. Practically we see only one particular realization so polarization is not zero but much less than the polarization of synchrotron radiation in a homogeneous magnetic field.

In a case of an anisotropic magnetic field with an axial symmetry which symmetry axis is perpendicular to the direction toward an observer and coincides with axis Ox (or Oy) the x and y field projections have different properties, $<B_{\perp y}^2> \neq <B_{\perp x}^2>$ so $<Q> \neq 0$ while $<U>=0$. If $B_{\perp}^2/B_{\parallel}^2$ is high then $<Q>$ can be close to the value for homogeneous magnetic field and polarization can be close to maximum theoretical limit $p=(\gamma+1)/(\gamma+7/3)$ for isotropic power law particle distribution function in homogeneous magnetic field.

\[
Q(\nu) = \frac{\sqrt{3}e^3}{mc^2} \int d\nu dE \frac{\nu}{\nu_c} B_{\perp}(\mathbf{r}) f_e(E, \mathbf{r}) K_{2/3} \left( \frac{\nu}{\nu_c} \right) \frac{B_{\perp y}^2 - B_{\perp x}^2}{B_{\perp}^2} \]

\[
U(\nu) = \frac{\sqrt{3}e^3}{mc^2} \int d\nu dE \frac{\nu}{\nu_c} B_{\perp}(\mathbf{r}) f_e(E, \mathbf{r}) K_{2/3} \left( \frac{\nu}{\nu_c} \right) \frac{2B_{\perp y} \cdot B_{\perp x}}{B_{\perp}^2}
\]
Model synchrotron radiation maps for photon energy 5keV, isotropic turbulence with different power spectrum index $\delta$.

The model synchrotron images are shown with the angular resolution 1.2". On each panel the total emission is in the upper left, the polarized emission is in the lower left, the polarization degree is in the upper right and the polarization angle (in radians) is in the lower right. Angle is measured from the Oy axis. The left and right panels respectively show an isotropic turbulence case for value of the turbulence power spectrum index $\delta=1;5/3$.

The dependence on the spectral index is seen as the relative sizes of the magnetic structures on the high resolution intensity and polarization images for different values of $\delta$. 
Stronger anisotropy in this geometry (shock compression case) results in a stronger polarization degree with polarization direction directed mostly radially.

Synchrotron radiation maps for photon energy 5keV, turbulence power spectrum index $\delta=5/3$ and anisotropic turbulence with $q=5; 20$. The model synchrotron images are shown with the angular resolution 1.2". On each panel the total emission is in the upper left, the polarized emission is in the lower left, the polarization degree is in the upper right and the polarization angle (in radians) is in the lower right. Angle is measured from the Oy axis. The left and right panels respectively show an anisotropic turbulence case with $q=5; 20$. The value of the magnetic turbulence power spectrum index is $\delta=5/3$.

Stronger anisotropy in this geometry (shock compression case) results in a stronger polarization degree with polarization direction directed mostly radially.
Synchrotron radiation maps for photon energy 5keV and the case of magnetic turbulence produced in cascading.

This picture presents the possible variation in images in case of different strength of magnetic turbulence. The radiation in a weaker turbulence case is more polarized. But the radiation in the simulated strong turbulence case is also significantly polarized. In all cases the most likely direction of polarization is along the Oy axis (tangent to the shock front) that differs from the case of turbulence produced in shock compression.

The angular resolution is 1.2". On each panel the total emission is in the upper left, the polarized emission is in the lower left, the polarization degree is in the upper right and the polarization angle (in radians) is in the lower right. Angle is measured from the Oy axis. The right panel shows the case of anisotropic turbulence produced in a weak cascade with q=1.05 and homogenous field component $B_{\text{hom}}^2 = \langle B_{\text{tur}}^2 \rangle$. The left panel shows the case of the same turbulence field without homogenous field component. This models a strong cascade case.
New generation of the X-ray polarimeters

XIPE (ESA) - artist's impression
(credits INAF-IAPS)

mission rejected

IXPE (NASA) - artist's impression
(credits NASA)

to be launched in 2020?

eXTP (Chinese Academy of Sciences) - artist's impression
(credits China Daily)

to be launched in 2025?
Gas Pixel Detector (GPD)

Simplified physics: The more probable direction of the photoelectron is along the electric field of the incoming X-ray photon. The projection tracks of photoelectrons are read out from the position sensitive anode.

Picture from INAF-IAPS (www.isdc.unige.ch/xipe/)
Model synchrotron radiation maps for photon energy 5keV after convolution with XIPE PSF, isotropic turbulence with different power spectrum index $\delta$.

The synchrotron images obtained after convolution with XIPE PSF (from S.Fabiani et al 2014) are shown. On each panel the total emission is in the upper left, the polarized emission is in the lower left, the polarization degree is in the upper right and the polarization angle (in radians) is in the lower right. Angle is measured from the Oy axis. The left and right panels respectively show an isotropic turbulence case for value of the turbulence power spectrum index $\delta=1; 5/3$.

The XIPE PSF width exceeds the smallest sizes of the image structure so after convolution the difference in small scale structure becomes much less prominent. From the other side the difference in the polarization degree still present after convolution so future X-ray polarimeter could give valuable information about the magnetic turbulence in SNR.
Synchrotron maps for shock compressed anisotropic turbulence with $q=5$ and magnetic turbulence produced in cascading with $q=1.05$.

The synchrotron images obtained after convolution with XIPE PSF are shown. On each panel the total emission is in the upper left, the polarized emission is in the lower left, the polarization degree is in the upper right and the polarization angle (in radians) is in the lower right. Angle is measured from the Oy axis. The left panel shows an anisotropic turbulence case with $q=5$. The right panel shows the model case of turbulence produced in a strong cascade with $q=1.05$ (left panel on fig. 8).

This picture presents the difference in observational properties of turbulence created due to shock compression or cascading processes. The polarization degree can be similar in both cases. But the most likely direction of polarisation differs significantly. This feature remains after convolution with XIPE PSF.
Significance estimation of the detected polarisation emission of Tycho SNR for the case of anisotropic magnetic turbulence formed by compression (q=5).

In the left there are synchrotron images obtained after the convolution with XIPE PSF. On each panel the total emission is in the upper left, the polarized emission is in the lower left, the polarization degree is in the upper right and the polarization angle (in radians) is in the lower right. In the middle the Tycho Chandra image is shown with the region used in simulations. In the right the model synchrotron emission and the significance of the detected polarization estimated as $S = \sqrt{Q^2 + U^2} / I$ is shown. The significance maps are binned with 10x10 arcsec pixels, the exposure time assumed to be 1 Ms.
Simulated significance maps for cases of anisotropic and isotropic magnetic turbulence.

Intensity and significance maps of the detected polarization estimated as $S = \sqrt{Q^2 + U^2/I}$. In the left a case of the anisotropic turbulence is shown ($q=5$), in the right a case of the isotropic turbulence is shown. Power spectrum index is $k=5/3$. Both maps are binned with 10x10 arcsec pixels, the exposure time assumed to be 1 Ms.
Significance simulation with XIMPOL.

Polarization, significance and intensity maps estimated with XIMPOL (ximpol.readthedocs.io). The case of the isotropic turbulence is shown. Power spectrum index is $k=5/3$. XIPE rmf and arf calibration files were assumed. Maps are binned with 20x20 arcsec pixels, the exposure time assumed to be 1 Ms.